# Space-time Blending

Prof. Pasko A.

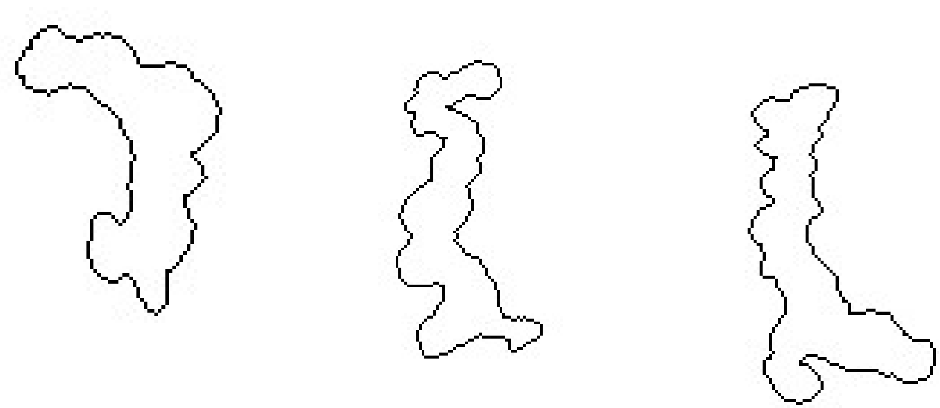
*http://hm.softalliance.net/*

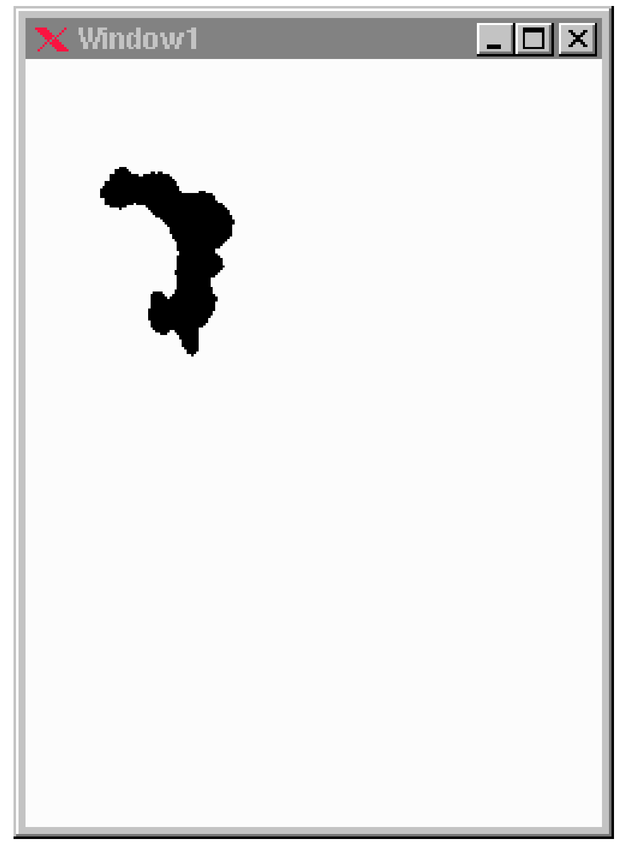
*Alexander Pasko, Evgenii Maltsev*

# Contents

- Approaches to 2D metamorphosis
- General transformation problem
- 3D blending
- Proposed approach
- Function representation FRep
- Bounded blending
- Transformation of 2D polygons

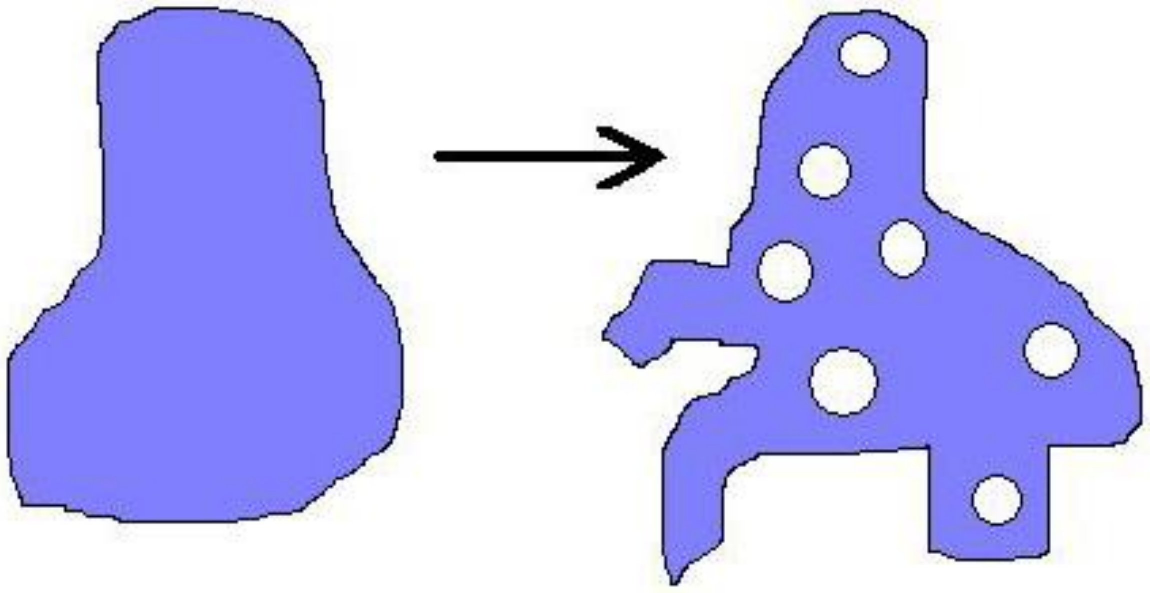
# Biological amoeba motion





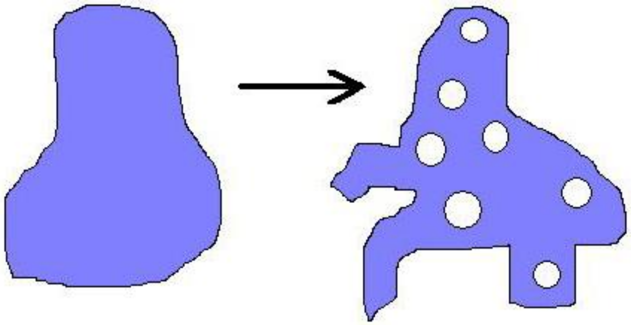

- Linear transformations (translation, scaling, rotation)
- Non-linear space mappings
- Topology change

# Metamorphosis



Initial shape    Final shape

# 2D metamorphosis



- Physically-based methods [Sederberg et al. 1992, Sederberg et al. 1993]
- Star-skeleton representation [Shapira  et al. 1995]
- Warping and distance field interpolation [Cohen-Or et al. 1996]
- Wavelet-based [Yuefeng Zhang  et al. 2000]
- Surface reconstruction methods [Surazhsky et al. 2001]

# Assumptions of existing approaches

- Equivalent topology (mainly topological disks)
- Shape alignment (common coordinate origin and significantly overlap in most of the case studies)
- Shape matching (establishing of shape vertex-vertex, control points or other features correspondence)
- Articulated figure motion
- Polygonal shape representation in most methods

# General transformation problem

- Arbitrary topology of initial 2D shapes
- Polygons, implicit curves, constructive solids, etc.
- No alignement or overlapping
- No correspondence between the boundary points or other shape features
- Combined transformation similar to biological amoeba motion

# Function representation FRep

A set of points with

$f(x,y) \geq 0$ is a 2D FRep solid;

Disk: $R^2-x^2-y^2 \geq 0$

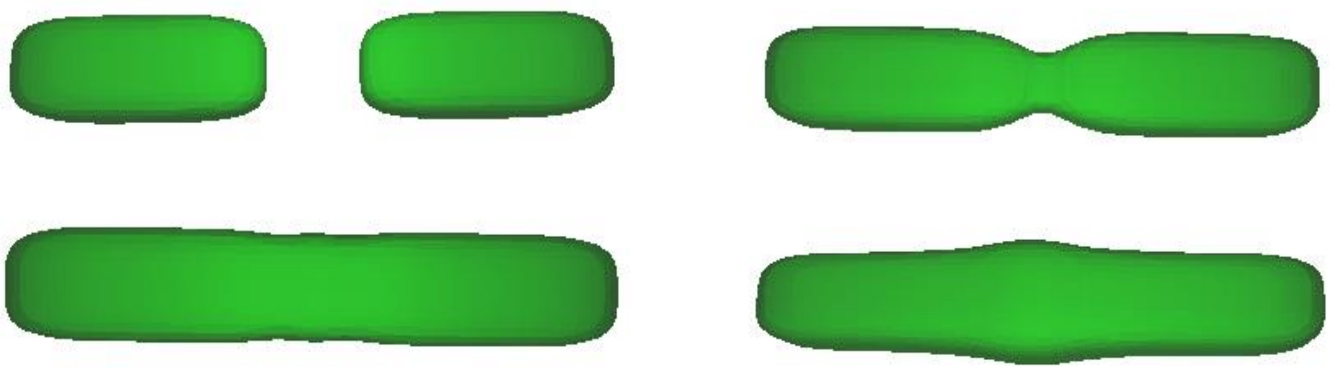
$f(x,y,z) \geq 0$ is a 3D FRep solid.

Solid ball: $R^2-x^2-y^2-z^2 \geq 0$

# Advantages:

- FRep solid can have arbitrary topology
- 2D polygonal shape can be converted automatically to a 2D FRep solid
- FRep solid can be constructed using primitives and operations
- Analytical formulations for set operations and blending

# 3D Blending



- Blending operation generates smooth transition between two surfaces.
- Usually used for modeling fillets and chamfers
- Blending union of two disjoint solids with added material - a single solid with a smooth surface
- Basis of our approach to the shape transformation

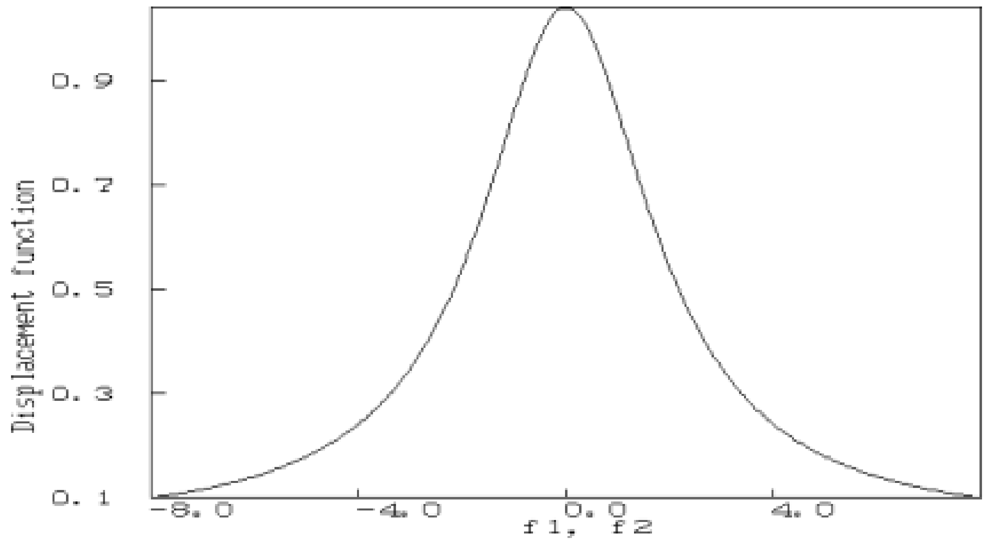
# Blending based on R-functions

$$F(f_1,f_2)= R(f_1,f_2) + d(f_1,f_2)$$

where *R* is an *R-function* - exact description of a set-theoretic operation (union),

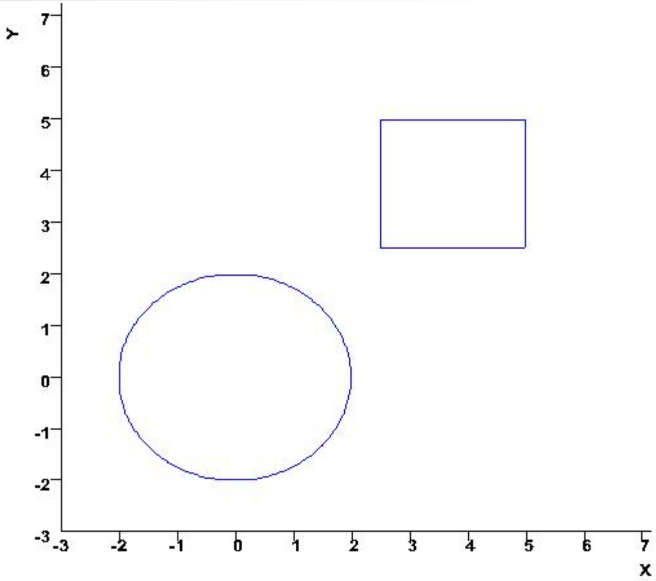
$$d(f_1,f_2) = a_0/(1+(f_1/a_1)^2+(f_2/a_2)^2)$$

The section $f_1=0$ for $d(f_1,f_2)$:

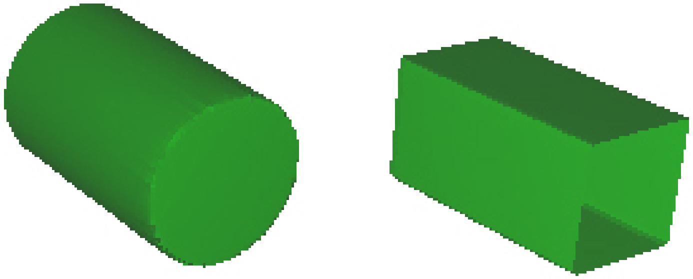


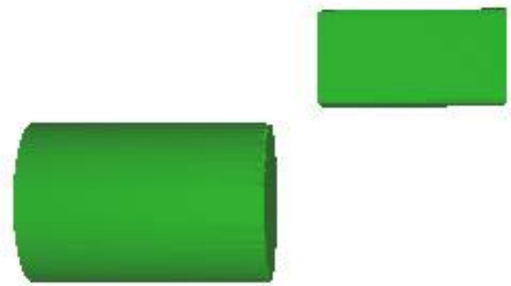
# Proposed approach

1. Two initial shapes on the *xy*-plane

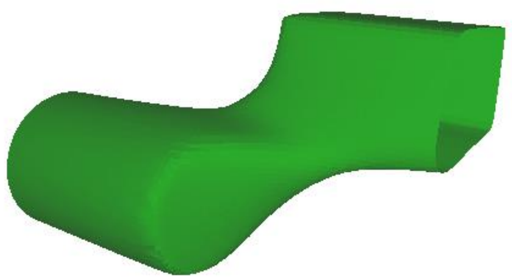



2. 2D shape as a cross-section of a 3D half-cylinder

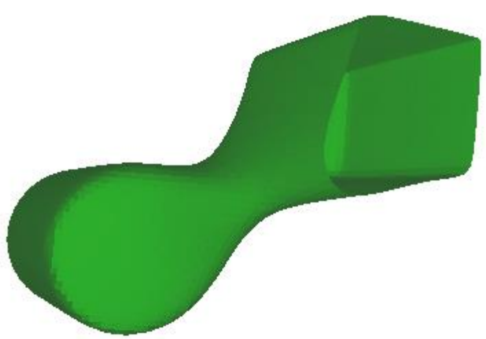




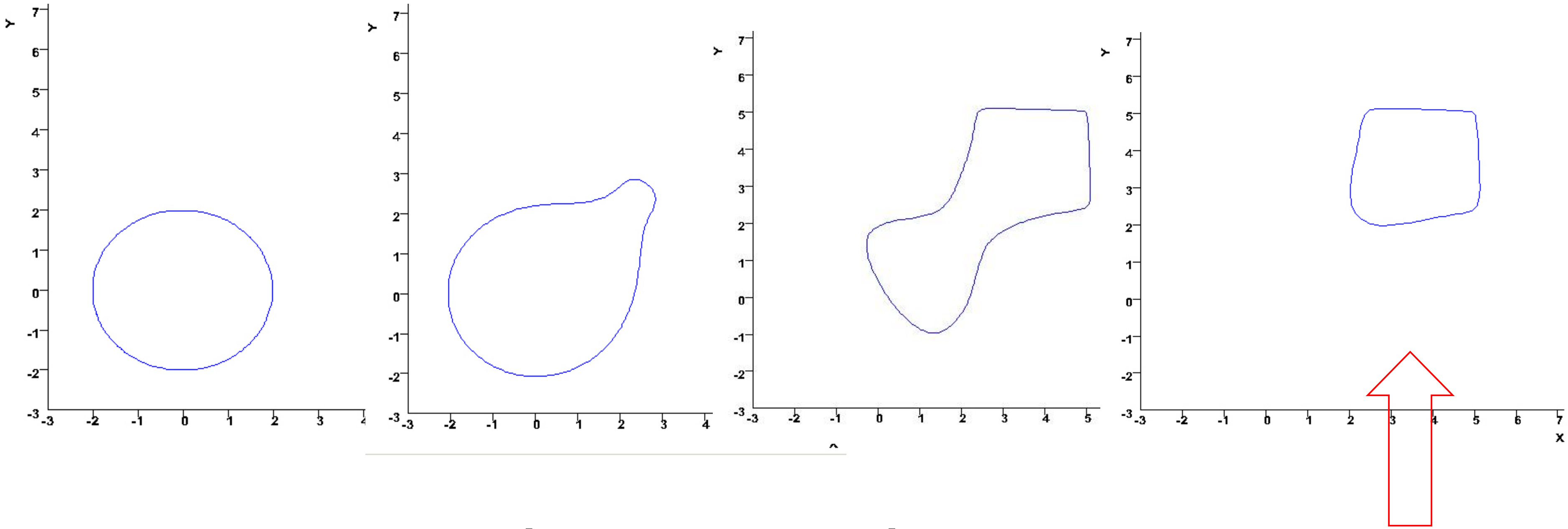
3. Leave gap for making the blend



4. Added material blend of the half-cylinders



5. Adjust parameters of blend to get satisfactory 2D cross sections




6. Consider *z* - coordinate as time and make 2D animation

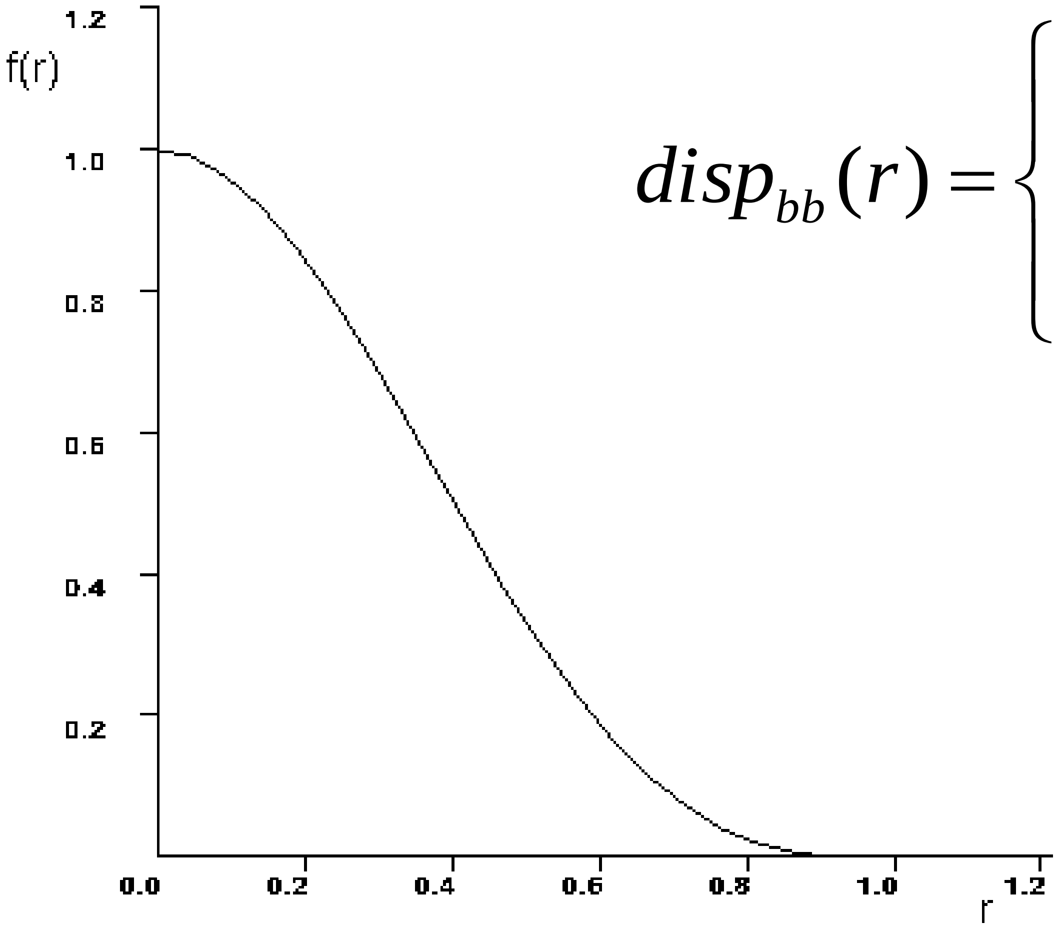
# Bounded displacement function

1) $disp_{bb}(r)$ takes the maximal value for $r=0$ ;

2) $disp_{bb}(r) = 0, r \geq 1$

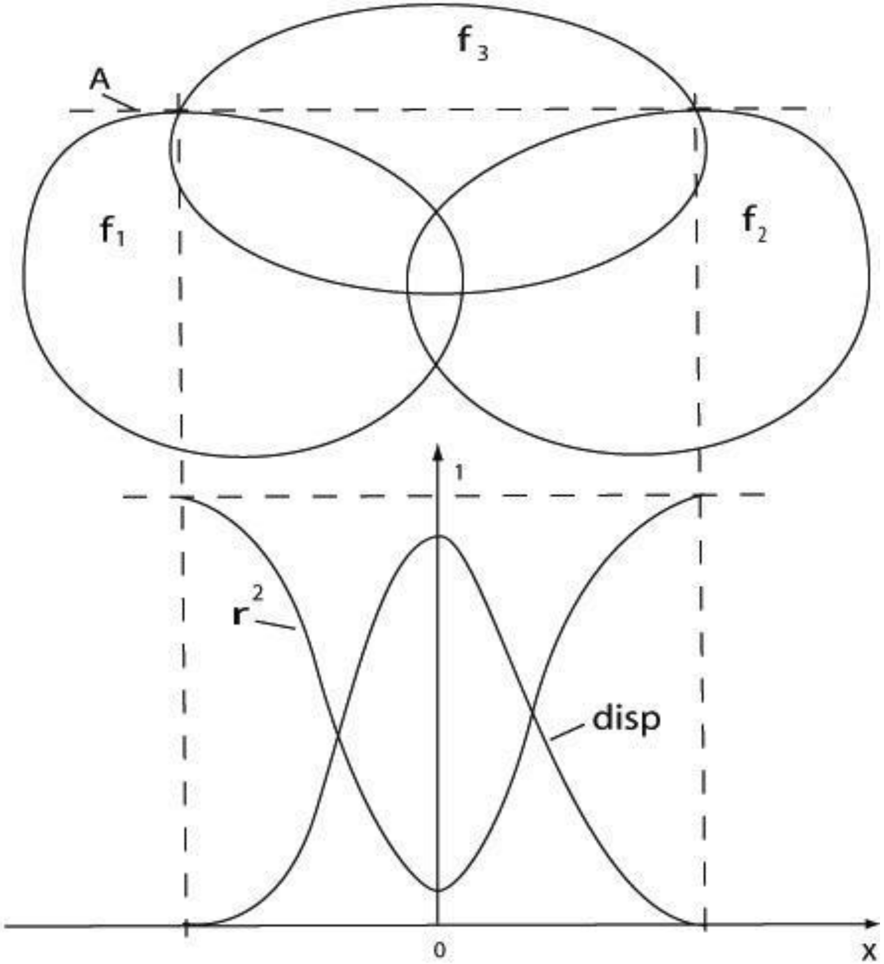
3) $\dfrac{\partial disp_{bb}}{\partial r} = 0, r = 1$ the curve tangentially approaches the axis at $r=1$.

# Bounded displacement function

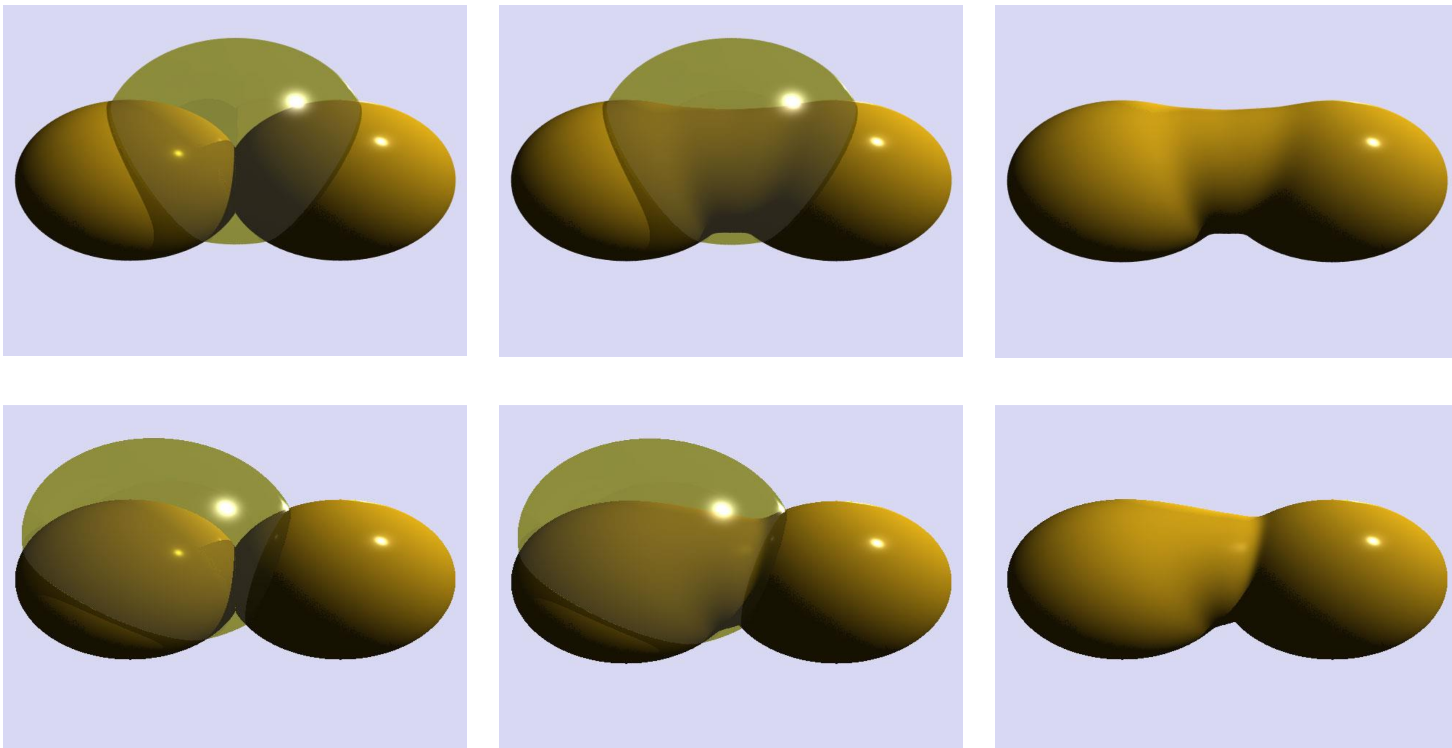



$$disp_{bb}(r) = \begin{cases} \dfrac{(1-r^2)^3}{1+r^2}, r < 1 \\ 0, r \geq 1 \end{cases}$$

# Bounding solid



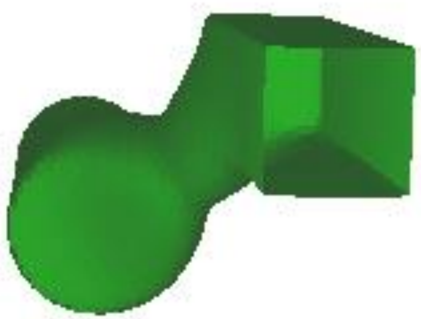
# Bounding solid



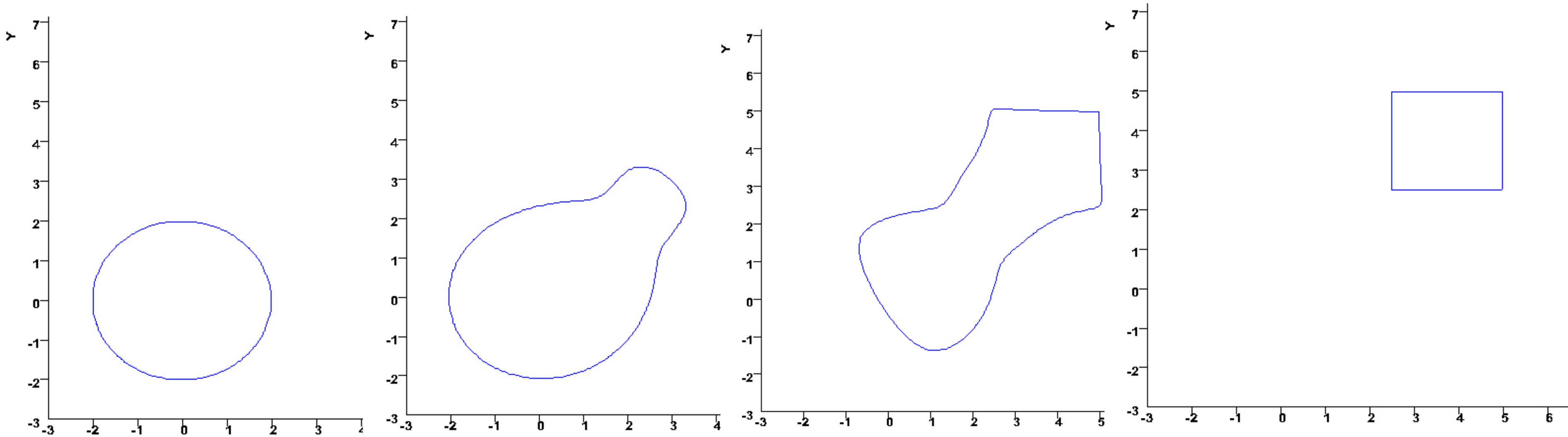
# Shape transformation using bounded blending

Bounded blending of two half-cylinders with a disk and a square as cross-sections and two bounding half-spaces

$z \geq -10$ and $z \leq 10$.

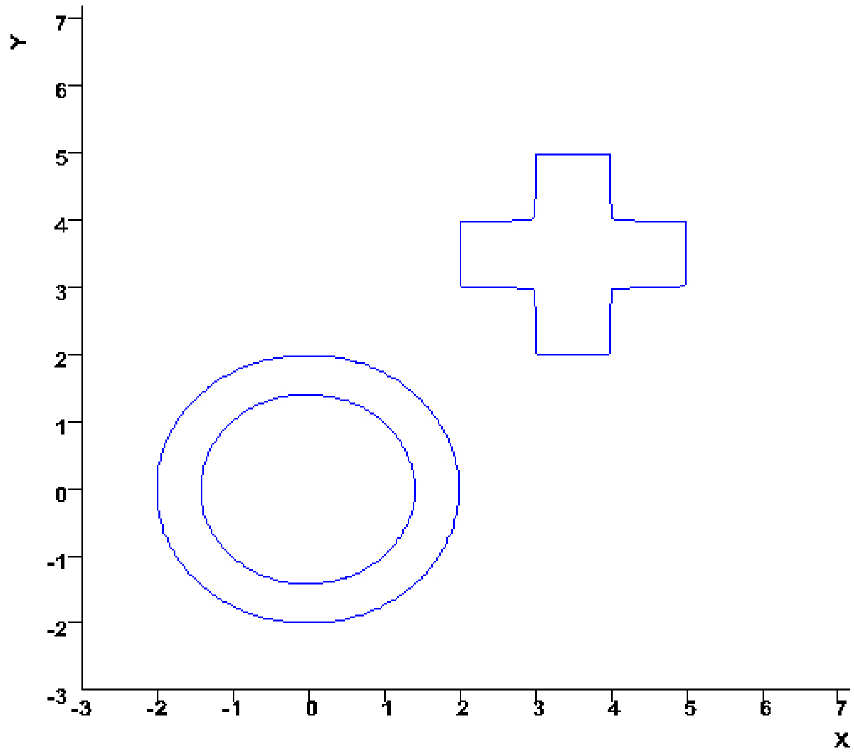


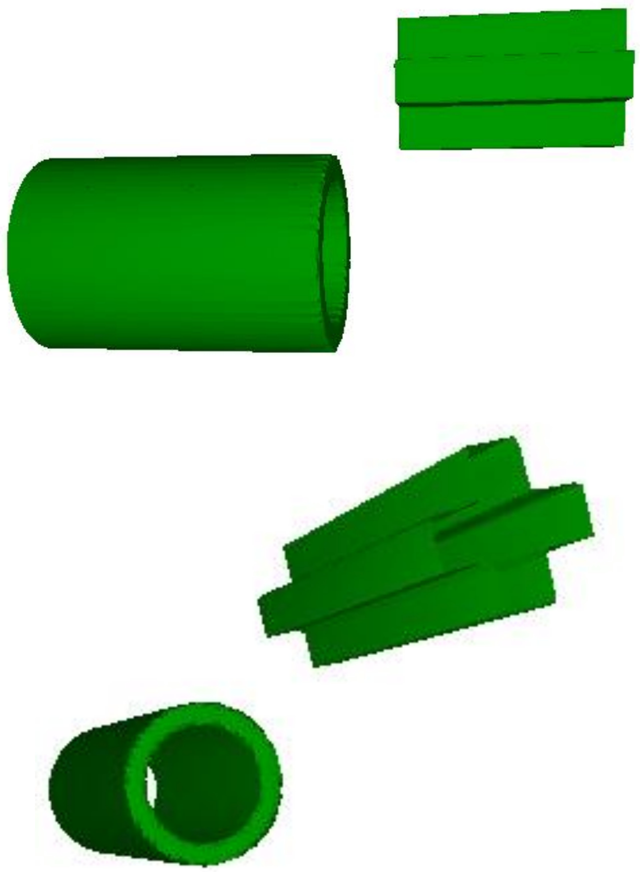
# Frames of animation: disk to square transformation using bounded blending




z=-10 z=0 z=1 z=10

# Ring to cross transformation

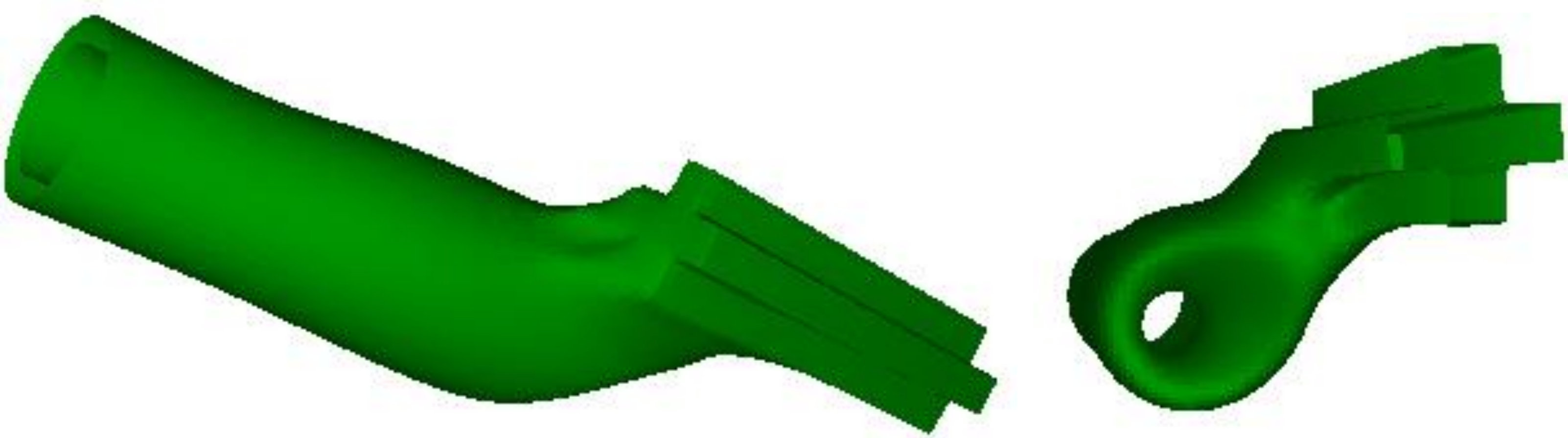


Initial 2D shapes

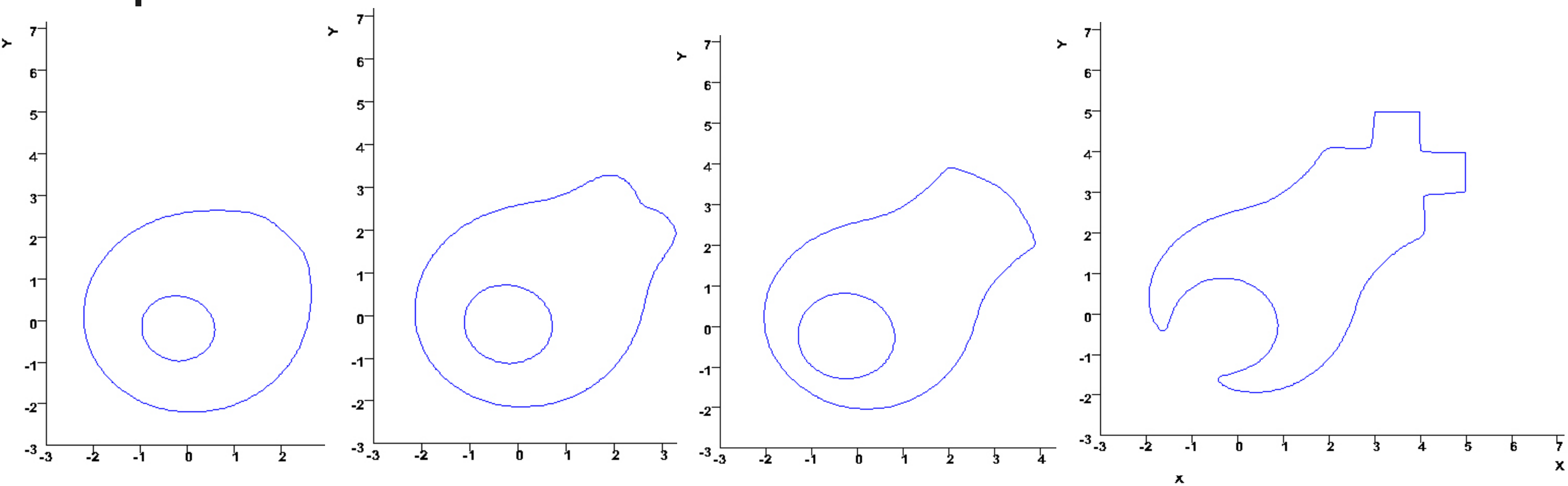
3D half-cylinders

# Ring to cross transformation



Bounded blending between 3D half-cylinders

# Ring to cross transformation




Frames of animation: ring to cross

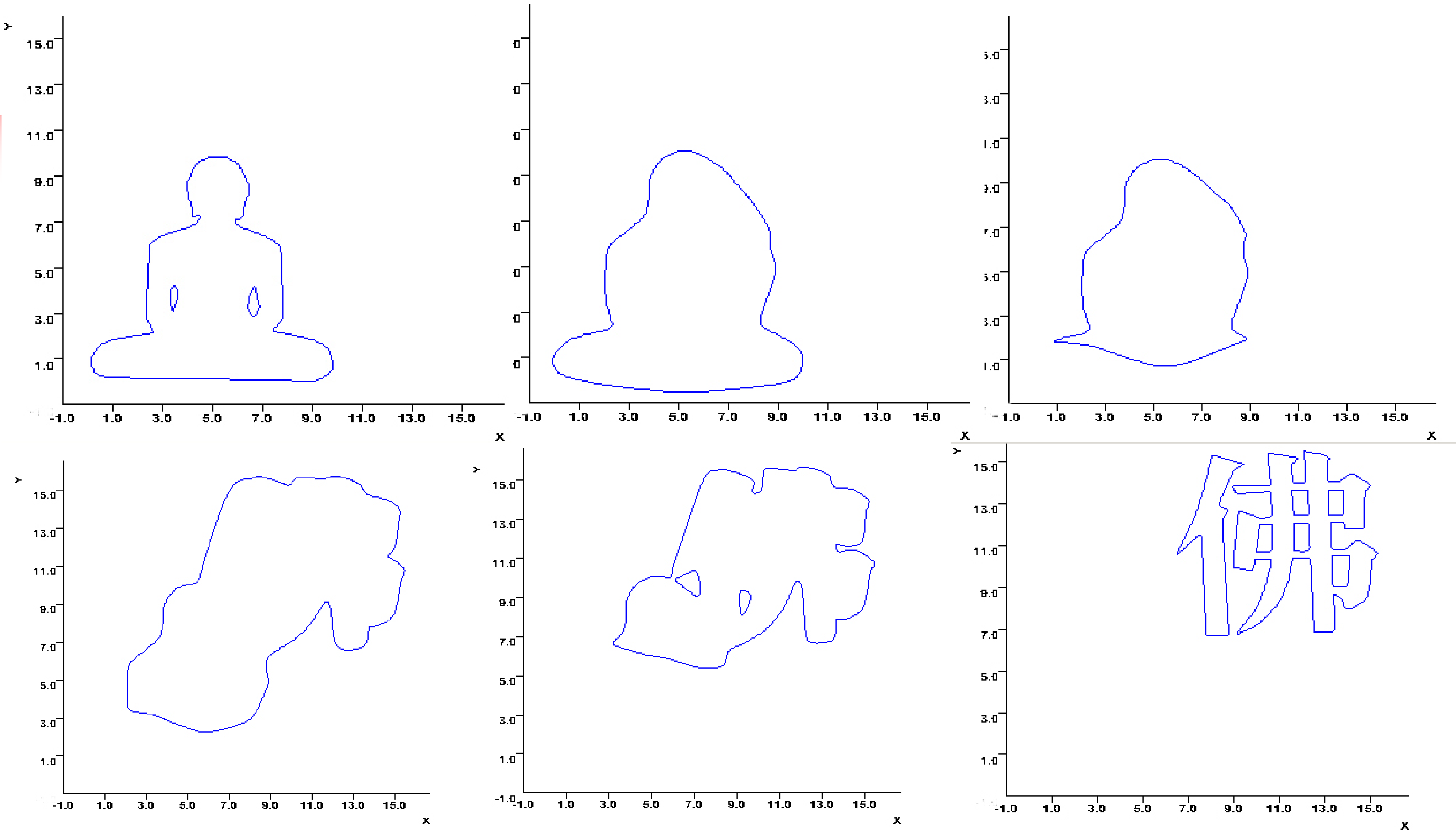
# Transformation of 2D polygons

Buddha shape and a Chinese character.

49 segments + 2 holes


2 disjoint components

# Transformation of 2D polygons




Frames of the animation: transformation of a Buddha shape into a Chinese character

# 3D metamorphosis

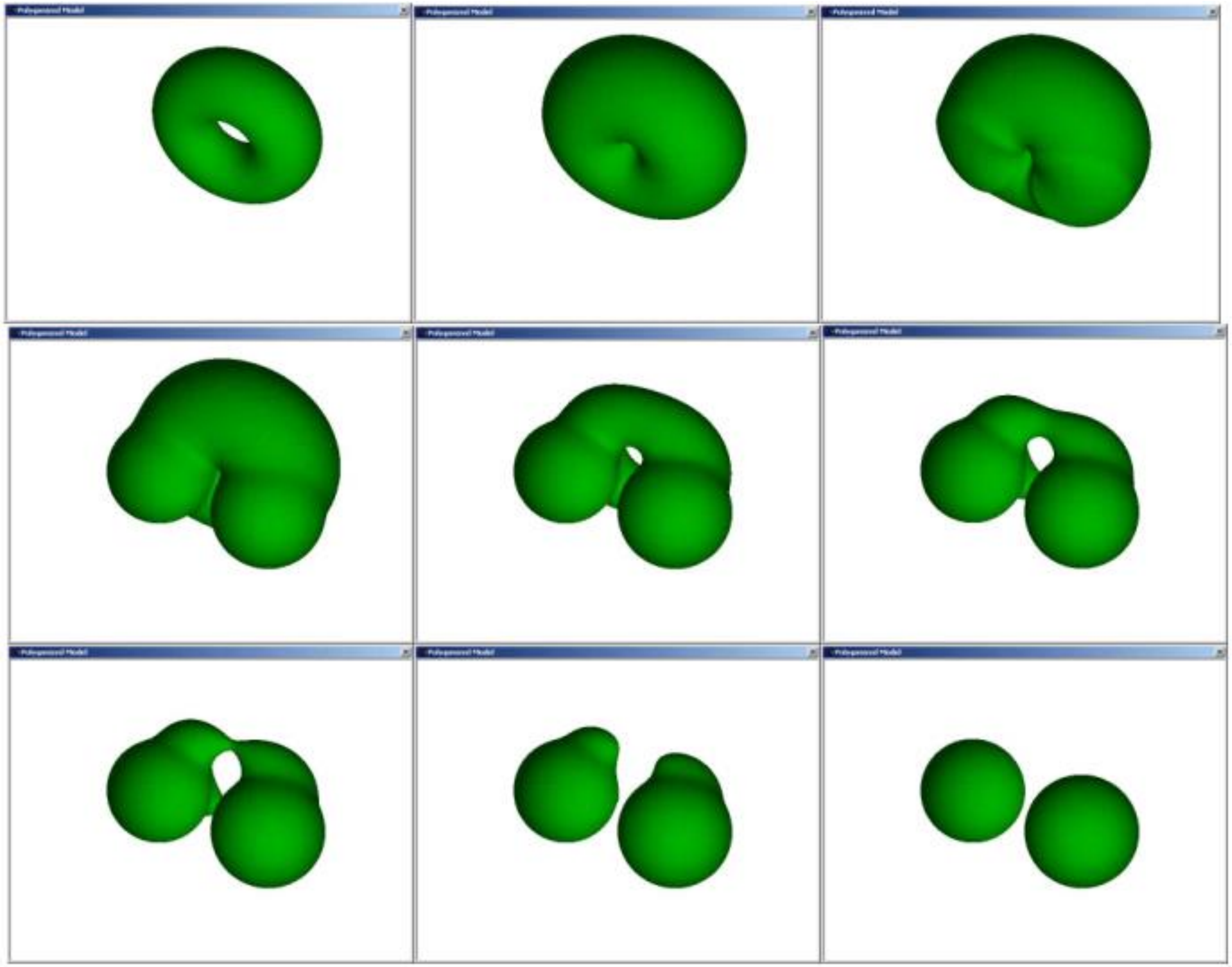
1) Initial 3D shapes in xyz-space;
2) Each shape is considered as a 3D cross-section of a half-cylinder defined in 4D space-time (a cylinder bounded by a plane from one side along the time axis);
3) The bounding planes of two half-cylinders are placed at some distance along time axis to provide an interval for making the blend;

# 3D metamorphosis

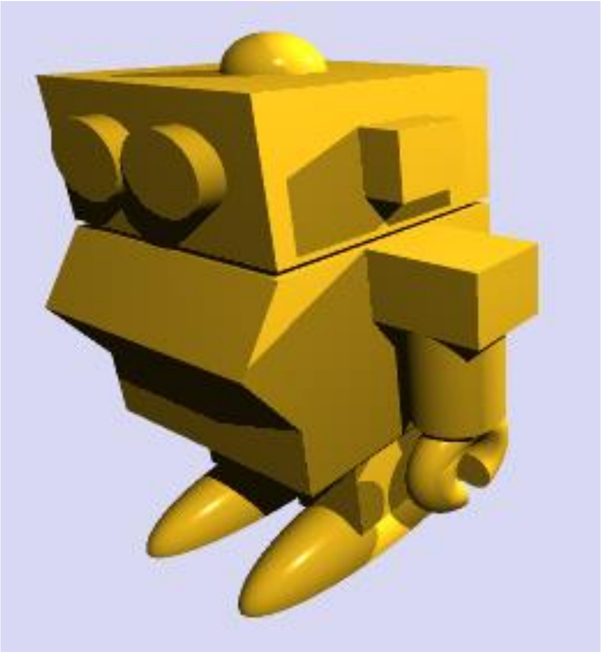
4) Apply the added material blending union operation to the half-cylinders;
5) Adjust parameters of the blend such that satisfactory intermediate 3D shapes are obtained in one or several cross sections along the time axis;
6) Make consequent orthogonal cross-sections along time axis and combine them into a 3D animation.

## 3D metamorphosis

- Tor to two spheres metamorphosis

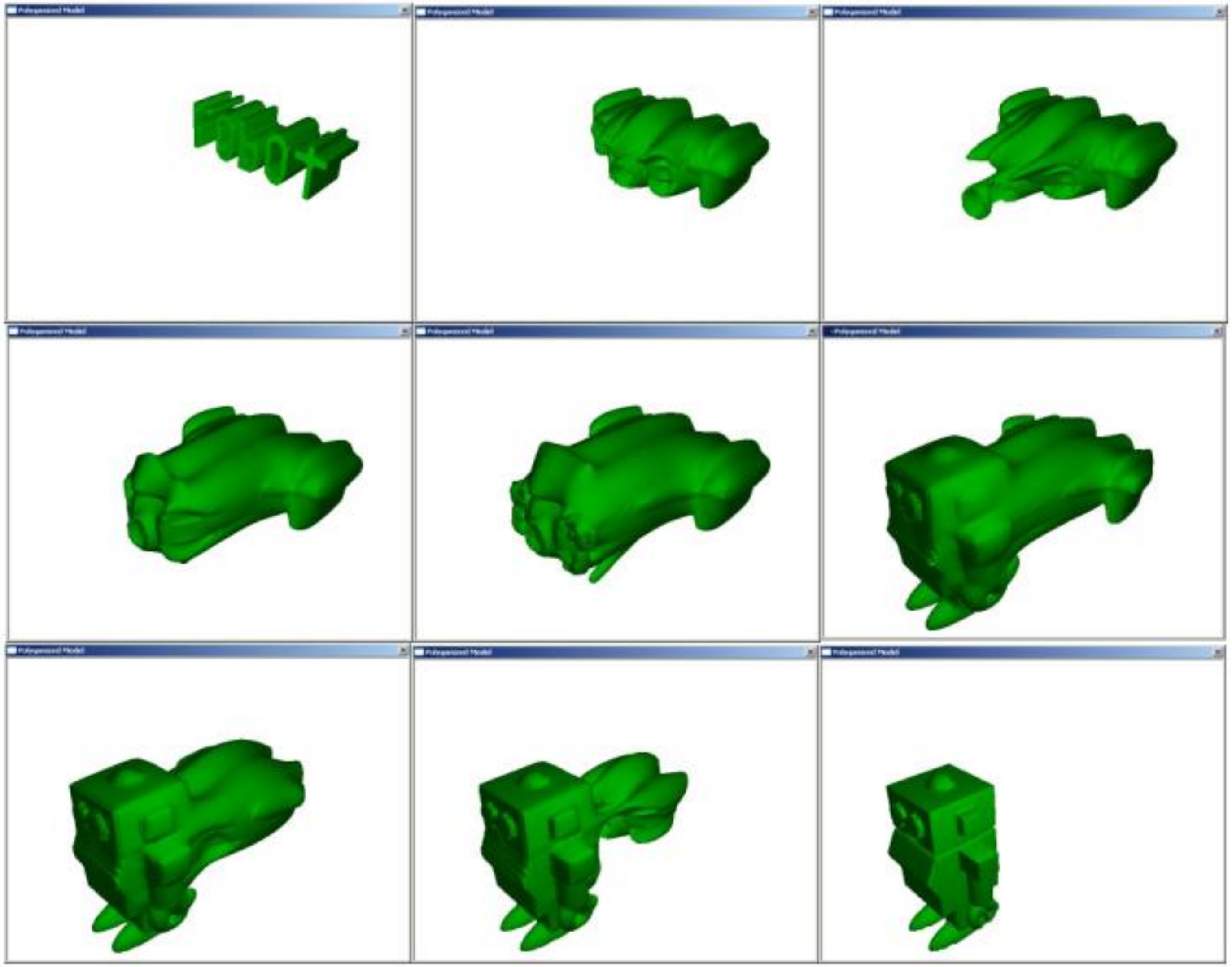


# 3D metamorphosis




Initial and final 3D shapes

# 3D letters to robot metamorphosis

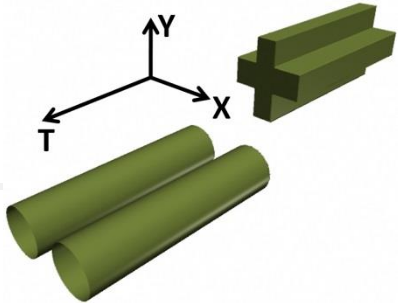


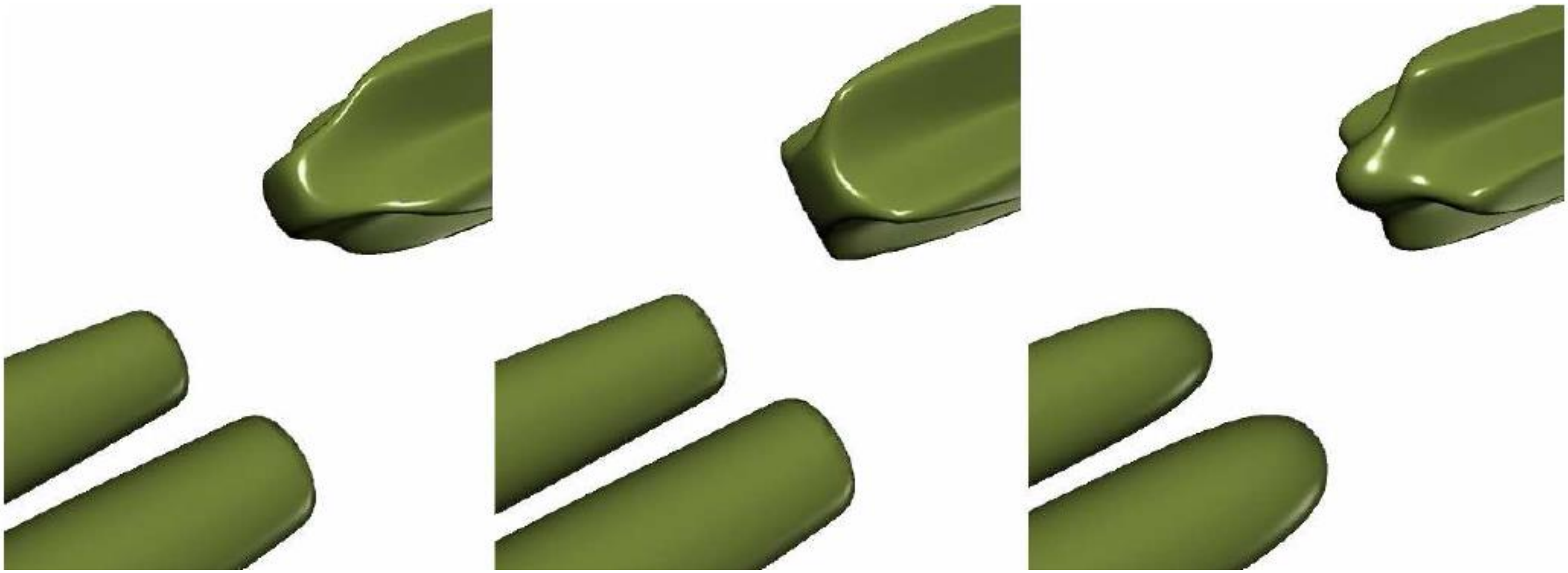
Frames of the animation

# Improved user control

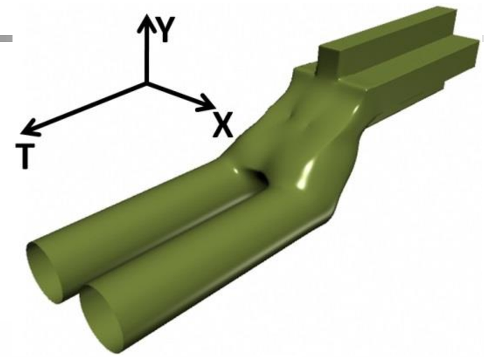
- Smoothed half-cylinders
- Additional affine and non-linear transformations using feature points
- Interactive editing
- Real-time rendering

# Smoothed half-cylinders



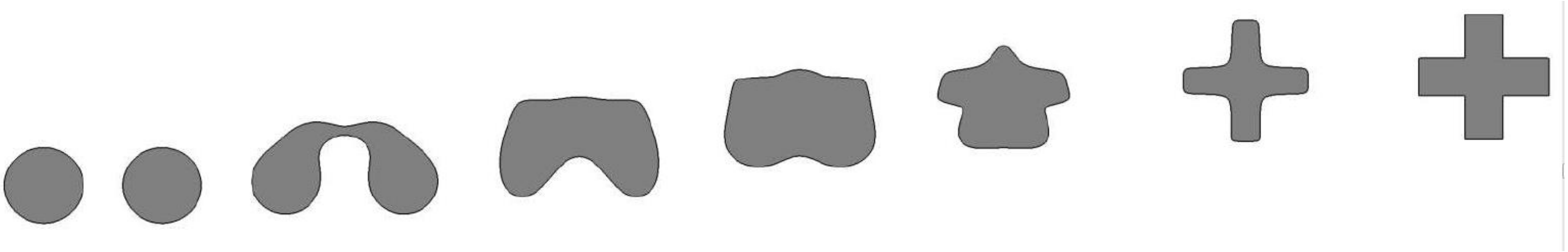
# Smoothed half-cylinders



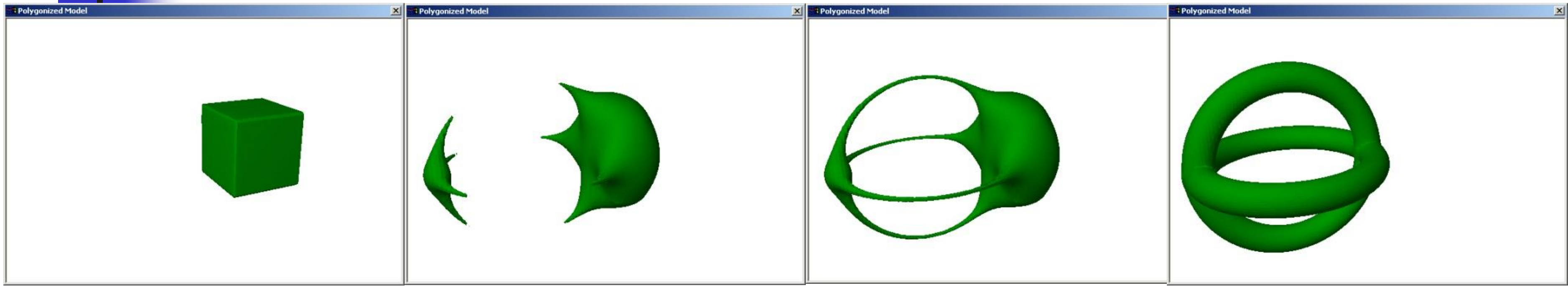

Added material blending of the smoothed half-cylinders

# Smoothed half-cylinders

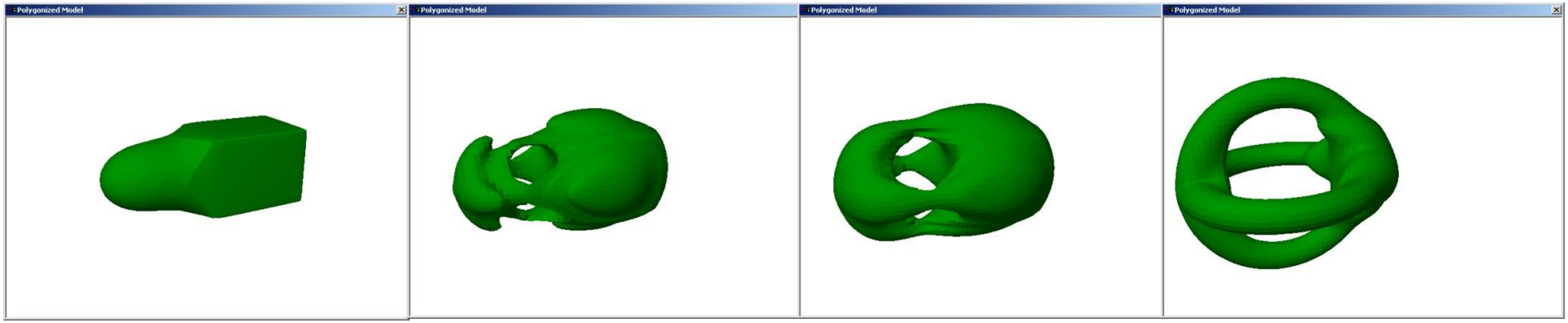
Consider $z$ - coordinate as time and make orthogonal slices



# Additional deformation



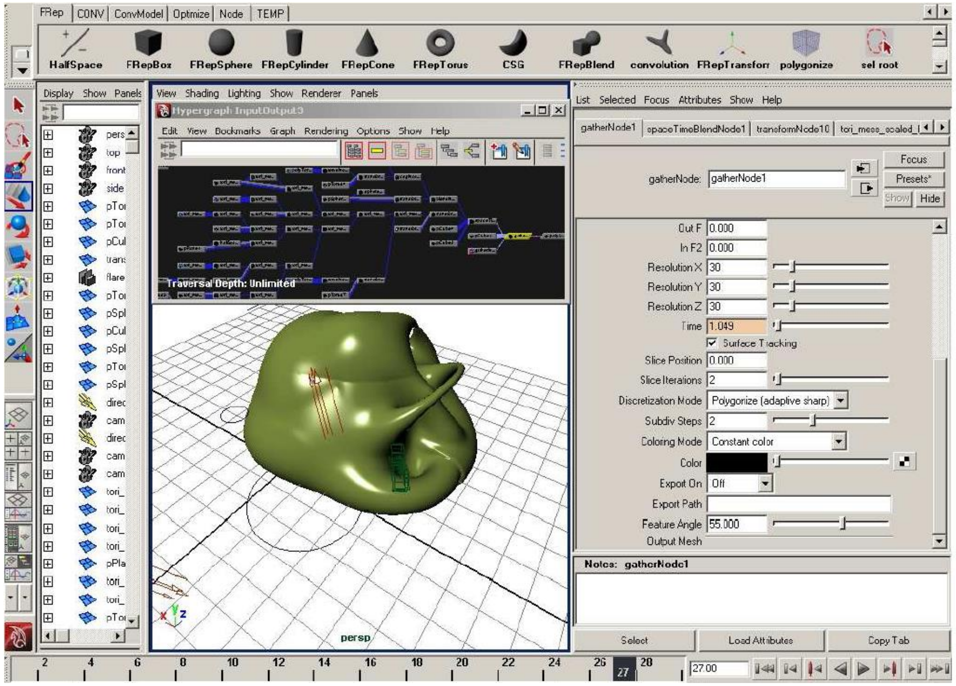

Metamorphosis with disjoint components




Additional non-linear mapping

# Interactive rendering and editing

Plug-in to Maya




Rendering in CUDA,

NVIDIA GeForce 8800

frames/second

| Grid resolution for polygonization | Ape to torii | Pumpkin to coach |
|---|---|---|
| **64x64x64** | 200 | 110 |
| **128x128x128** | 60 | 30 |

# Conclusions

- New approach and analytical formulations for 2D and 3D metamorphosis on the basis of
  - dimension increase from k to k+1
  - bounded blending between (k+1)D objects
  - cross-sectioning the blend area
  - getting frames of the animation

# Conclusions

- Proposed approach can handle non-overlapping shapes with arbitrary topology
- Amorphous or amoeba-like behaviour including non-linear motion and metamorphosis.
- The case of 3D input shapes is processed using bounded blending of 4D space-time objects.

# References


- G. Pasko, A. Pasko, T. Kunii
  Space-time blending, Computer Animation and Virtual Worlds, vol. 15, No. 2, 2004, pp. 109-121.